A Decomposition Approximation for the Performance Evaluation of Non-Preemptive Priority in GSM/GPRS

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Abstract

Decomposition Approximations have in the past been successfully applied to the performance analysis of Preemptive Priority based GSM/GPRS networks. In this paper we propose a decomposition technique for the performance analysis of GSM/GPRS networks where GSM voice calls have non-preemptive priority over GPRS data packets. In other words when demand exists for GSM circuit switched channels the GPRS user can continue its transmission until the ending of packet transmission resulting in a delayed release of the channel. It is shown that such an approximation can be quite accurate in predicting both the impact of delayed release on GSM voice queuing as well as for comparison of immediate versus delayed release of channels on GPRS data queuing delays.[1]

1. Introduction

A number of analytical techniques have been used in the past for the performance analysis of GPRS, where data packets share channel resources with GSM voice calls. Ni and Haggman [1] introduced the use of the decomposition method for analysis of GPRS networks, based on Ghani and Schwartz [2]. This method is based on the fact that holding times for GSM circuit switched services are much longer than the time required for transmitting GPRS packets (see also [9], [10]). Two alternatives may further exist termed as *Immediate Release* and *Delayed Release* as described in [1] and [3]. In the former case, the GPRS user is forced to stop transmission immediately upon the arrival of circuit switched calls, whereas in the latter case, the GPRS user is allowed to continue transmission until completion of packet transmission, and GSM voice calls would require queuing due to the delayed release of channels that are occupied by GPRS data packets. Hence in the former case GSM voice has preemptive priority over GPRS data, whereas in the latter case GSM voice has non-preemptive priority over GPRS data. Most of the analytical analyses of GPRS such as [1], [4] - [8] have assumed the preemptive priority, Immediate Release alternative. Due to the intractable nature of the non-preemptive priority analysis, wherever this alternative has been analyzed as in [3] and [6] it has been done via simulation only and no analytical model has been developed. In [7] and [8] the case with no channel de-allocation has been analyzed implying that no queuing of GSM voice calls is allowed as required in the delayed release case.

In this paper we propose to solve the non-preemptive priority (Delayed Release) model, as we believe that an analytical model for this has not been developed, and it may be useful to analyze this case as well. Hence we develop an approximation model for analyzing the Delayed Release alternative, for single-slot GPRS, and propose a new technique based on the decomposition approximation introduced in [2]. In this scenario circuit switched GSM voice calls have *non-preemptive priority* over the GPRS data and hence GSM voice calls experience both blocking and queuing.

2. System Model

We assume that N physical transmission channels are available in the system. Of these N_d are dedicated for GPRS data, and $N_v = N - N_d$ are shared by circuit switched GSM voice and GPRS data, with GSM voice having non-preemptive priority over GPRS data. If the number of GSM voice calls in the system is less than N_v, then a new call is accepted, i.e. it is not blocked. After being accepted, if one or more of these N_v channels are free then the call is transmitted. If on the other hand, none of the N_v channels are free, implying that the number of GPRS data packets in service is greater than N_d, then the call is queued, waiting for a GPRS data packet or GSM voice call to complete service.

The arrival process of voice and data are approximated by a Poisson arrival process, with average arrival rates of λ_v, λ_d respectively, and the holding times, neglecting the granularity of the frame structure, are approximated by an exponential distribution, with average holding time of μ_v, μ_d respectively. Further,

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we also assume that the queued GSM voice calls do not wait endlessly on queue, but instead stay in the queue for a random amount of time τ. If they are not served by the end of this time period then they leave the queue, i.e. the caller hangs up. In the following analyses, we make the simplifying assumption that τ is exponentially distributed with mean equal to the holding time of a call, i.e. $E(\tau)=1/\mu_v$.

The system described above can be modeled by a three dimensional Markov chain $\{V(t),D(t),D'(t)\}$, whose steady state probabilities $P_{i,j,j'}$ are desired to be found:

$V(t)=i=$ Total number of circuit switched GSM voice calls, $i=0,\cdots,N_v$;

$D(t)=j=$ Total number of GPRS data packets in the system, $j=0,\cdots,M$;

$D'(t)=j'=$ Number of GPRS data packets in service, $j'=0,\cdots,N$.

If we let $V_Q(t)$ be the number of queued GSM voice calls at time t, then $V_Q(t)$ is given by

$$V_Q(t)=\max\{0,V(t)-(N-D'(t))\}. \qquad (2.1)$$

Note that $V(t)$ can only take values in the range 0 to N_v, hence for $D'(t)<N_d$, $V_Q(t)$ is always zero. We also define M as the maximum number of GPRS data packets that are allowed in the system. We henceforth use (i,j,j') as a shorthand notation to denote the state $(V(t)=i,D(t)=j,D'(t)=j')$.

Now the set of possible states S, of the Markov chain $\{V(t),D(t),D'(t)\}$ is given by:

$$S\equiv\{(i,j,j'):0\le i\le N_v;0\le j\le M;\min(j,N-i)\le j'\le\min(j,N)\}. \qquad (2.2)$$

The minimum value of j' for $(i,j,j')\in S$ occurs when none of the GSM voice calls are queued. i.e. $V_Q(t)=0$, and the maximum occurs when GPRS data is occupying as many GSM voice channels as possible. The following is an example of the notation that we shall henceforth use for denoting a subset of S:

$$\{1,j\ge N,N\}\equiv\{(V(t),D(t),D'(t)):V(t)=1,D(t)\ge N,D'(t)=N\}.$$

Figure 1 shows the state transition diagram for $N=3$, $N_v=2$, $N_d=1$ and $M=5$. Using the $N(N_v,N_d)M$ shorthand notation of [2], this system can also be described as a *3(2,1)5 Non-preemptive GPRS* system.

We now partition the state space S as follows. Let S_Q be the set of *Queued states*, i.e. those in which one or more GSM voice calls are queued $(V_Q(t)>0)$. These are shown with the solid circles in figure 1. The rest of the states in S are denoted by S_N, the set of *Non-queued states*, i.e. the states in which no GSM voice calls are queued. These are shown as the solid dots in the figure. Using figure 1, one may now write down all possible transition rates (as done in the Appendix).

3. GSM Voice call analysis

Since we have assumed that queued GSM voice calls hang up at the same rate as the rate at which calls in transmission are completed, the total rate at which GSM voice calls leave the system depends only on $V(t)$. The GSM voice arrival process is already independent of the GPRS data process. This implies that the *total* number of GSM voice calls in the system $V(t)$ is independent of the GPRS data process and the equilibrium distribution of $V(t)$ is simply that of an $M/M/N_v/N_v$ queuing system which is given by the Erlang-B formula:

$$p_v(i)=\frac{\rho_v^{\,i}}{i!}\Big/\sum_{j=0}^{N_v}\rho_v^j/j! \quad \text{for} \quad i=0,1,\ldots,N_v,$$
$$\text{where } \rho_v\equiv\lambda_v/\mu_v. \qquad (3.0)$$

The GSM voice blocking probability, i.e. the probability that an arriving GSM voice call is not accepted into the system is:

$$P_{BV}=p_v(N_v)=\frac{\rho_v^{N_v}}{N_v!}\Big/\sum_{i=0}^{N_v}\rho_v^i/i! \qquad (3.1)$$

But now in addition to GSM voice blocking, we also have GSM voice queuing, and $V_Q(t)$ the number of queued GSM voice calls *does* depend on the GPRS data process. Thus in order to determine the queuing delay for GSM voice, we need to solve the complete Markov Chain $\{V(t),D(t),D'(t)\}$. This is of course also necessary to evaluate the GPRS data performance since $D(t)$ and $D'(t)$ clearly depend on $V(t)$.

Due to the existence of unidirectional transitions, as shown in figure 1, it is clear that the Markov chain $\{V(t),D(t),D'(t)\}$ is not reversible and that a simple closed form solution does not exist. The problem also has greater complexity than the equivalent preemptive problem, solved in [2], due to the added dimension in the Markov chain by the queued GSM voice states.

In order to find an approximate solution to the problem, we apply the decomposition technique to the problem. We make the underlying assumption of [2] that α defined as the ratio of GSM voice holding times to that of GPRS data is large, i.e.:

$$\alpha\equiv\frac{1/\mu_v}{1/\mu_d} \gg 1.$$

This is based on the fact that typical GSM voice calls occupy channels for 120 – 180 seconds, as compared to GPRS data packets which are typically transmitted within 2 – 10 seconds (see [1]).

The decomposition technique essentially assumes that the steady state behavior of such systems can be approximated by converting the multi-dimensional Markov chain into a hierarchy of group of *aggregate states*, such that the interaction *between* the groups is small compared to the interaction *within* the groups. A group of states would thus comprise all the states for a fixed number of GSM voice calls. Hence for the duration of a GSM voice call, the technique assumes that the GPRS data process achieves steady state, and hence its equilibrium distribution may be approximated by ignoring the transitions between groups.

4. Overview of the Approximation

In the following sections we first present an overview of the overall technique used to find an approximate solution using the 3(2,1)5 example of figure 1. The detailed analysis appears in Appendix A.

The first step in the decomposition approximation is to identify the group of *aggregate states*, as defined in [2], section III. In this case, as explained above, it is easy to see that these are the states for which $V(t)$ is constant. In figure 1 these are the states circled with a dotted line corresponding to the three aggregate states $V(t)=0,1$ and 2. We note that all transitions *between* aggregates are in terms of λ_v and μ_v, whereas all transitions *within* aggregates are in terms of λ_d and μ_d. We also note that since the equilibrium distribution of $V(t)$ is already known, as given by (3.0), the probability of being in an aggregate state is known exactly.

The next step in the decomposition technique is to find the decomposition solution to each one of the states within each aggregate state in isolation, i.e. after removing all the transitions *between* the aggregate states. In the following description, the labeling '(step I, part A/B)' refers to the steps in Algorithm 1, described later in Appendix A.

Decomposition for V(t) = 0.

Step 0, Part A. (figure 2.0.A). For the aggregate state $V(t)=0$, consisting of states $\{0, j \geq 0, j' \geq 0\}$, after isolation we get a simple $M/M/N/M = M/M/3/5$, birth-death process whose solution is easily found.

If we now proceed via a simplistic application of decomposition to aggregate state $V(t)=1$, then we note that after elimination of all the transitions between aggregate states, the states belonging to S_Q, i.e. states $(1,3,3),(1,4,3),(1,5,3)$, become transient states since there are no longer any transitions from S_N to S_Q, but only transitions from S_Q to S_N. Hence the decomposition solution would give $P_{i,j,j'}=0$, $\forall(i,j,j') \in S_Q$, which would give a zero queuing delay for the GSM voice process. This would give a grossly inaccurate result, and so we must modify our technique.

We observe that for all the states in $\{i \geq 1, j \geq N, j' = N\}$, i.e. states (1,3,3), (1,4,3), (1,5,3), (2,3,3), (2,4,3), and (2,5,3) in figure 1, the transitions into these states are only from the states in $\{i = 0, j \geq N, j' = N\}$, i.e. states (0,3,3), (0,4,3), (0,5,3) in figure 1. This fact is used in Proposition 1 of the Appendix to find an exact dependence of the equilibrium probabilities of states in $\{1, j \geq N, N\}$ on those of states in $\{0, j \geq N, N\}$. But the equilibrium probabilities of the states $\{0, j \geq N, N\}$ were already approximated in step 0 above. Hence we can obtain an approximation to $P_{i\geq 1, j \geq N, N}$ using the approximate solution to $P_{0, j \geq N, N}$. The inaccuracy in $P_{i\geq 1, j \geq N, N}$ would be directly related to the inaccuracy in $P_{0, j \geq N, N}$ since the relationship between $P_{i\geq 1, j \geq N, N}$ and $P_{0, j \geq N, N}$ can be found exactly. Thus we have

Step 0, Part B. (figure 2.0.B). Obtain solution to $P_{i\geq 1, j \geq N, N}$ using exact dependence on $P_{0, j \geq N, N}$ (shown later in Proposition 1), whose solution was obtained earlier in step 0, Part A.

Decomposition for V(t) = 1.

Step 1, Part A. (figure 2.0.A). Having approximated $P_{i\geq 1, j \geq N, N}$ we can now find a decomposition solution to the rest of the aggregate states in $V(t)=1$, by first isolating the aggregate states. We are then left with the rather unusual Markov chain shown in figure 2.1.A in which the equilibrium distribution to part of the Markov chain is already known. We can now solve for the equilibrium distribution of the remaining unknown states by writing the local balance equations at all the states $(i,j,j') \in S_N$ using the known equilibrium probabilities $P_{1,j,N}$. We thus find an approximate solution to the equilibrium probabilities of all the states in aggregate state $V(t)=1$. The details are developed in the Appendix.

Step 1, Part B. (figure 2.1.B). The procedures used in step 0, part A can essentially be repeated to find all the equilibrium probabilities for states $\{i \geq 2, j \geq 2, N-1\}$, i.e. (2,2,2), (2,3,2), (2,4,2), and (2,5,2). The only difference is that in addition to using the known equilibrium probabilities of states (1,2,2),…,(1,5,2), we also use those of states (2,3,3), (2,4,3), and (2,5,3) which were approximated in step 0, part B.

Decomposition for V(t) = 2.

Step 2, Part A. (figure 2.2.A). Finally we approximate the equilibrium probabilities of states (2,0,0), (2,1,1), … (2,5,1) by using the known equilibrium probabilities of states (2,2,2), … (2,5,2) by a procedure similar to step 1, part A.

Having approximated all the equilibrium probabilities we can now use these to find the performance criterion of interest. In addition to the GPRS data delay and blocking probability, we can now also find the GSM voice average queue length and waiting time in queue.

A detailed analysis of the decomposition approximation appears in Appendix A. The analysis comprises of three parts. In the first, we develop an exact relationship between the equilibrium probabilities of queued states S_Q in terms of the equilibrium probabilities of non-queued states S_N, using a recursive solution given by Proposition 1. In the second part we find a decomposition solution to the states within each aggregate state, based on all the known equilibrium probabilities. Finally an algorithm is given which calculates all the equilibrium probabilities in the correct order.

5. Results and Conclusions

Several results are presented in figures 3 – 7. In all cases simulation results are presented alongside the decomposition approximation for validation of the approximation. The main usefulness in the approximation is to estimate the impact of the delayed release of GPRS data channels on GSM voice call queuing.

Figure 3 shows the average GSM voice queue length versus GPRS data utilization ($\rho_d \equiv \lambda_d / \mu_d$), for the 2(1,1)30 non-preemptive GPRS system, i.e. with total channels N=2; dedicated channels for GPRS data N_d=1; shared GSM/GPRS channels N_v=1; and the maximum number of GPRS data packets that are allowed in the system M=30. Here two curves are shown for two different values of α, the ratio of GSM voice holding time to that of GPRS data: α = 10 and 100. The GSM voice utilization is fixed at $\rho_v = 0.5$. The results indicate that as we increase α, the decomposition approximation becomes very accurate in predicting the GSM voice queue length (note that for α=100 the two curves are essentially superimposed). We also note that the decomposition approximation slightly exaggerates the GSM voice queuing delays. Figure 4 and 5 show similar results for larger systems: 8(4,4)100, with ρ_v= 2.5, and α = 100 and 300; and 50(25,25)100, with ρ_v= 25.3, and α = 300 and 1000 respectively[2].

We note that unlike the previous decomposition approximations in [2], the decomposition approximation for Non-preemptive priority does depend on α. We also note from both these figures that while keeping all other factors constant, as we increase α, the mean GSM voice queue length decreases. This is what one might expect, since if the GSM voice calls take longer to complete as compared to GPRS data, the proportion of time that a call has to wait for GPRS data reduces.

Figure 6 compares the Non-preemptive priority results with Preemptive priority for the 2(1,1)30 system studied above, with ρ_v= 0.5. As expected we note that the GPRS data queues are marginally shorter for Non-preemptive priority systems as compared with preemptive priority systems. We see here that the decomposition approximation is useful in depicting the difference between the preemptive priority vs. non-preemptive priority schemes. Figure 7 shows the mean GSM voice waiting time EW_V, for the 8(4,4)100 non-preemptive priority GPRS system, with the mean GPRS data holding time $1/\mu_d$ = 2 seconds. Here again we note that the approximation is quite useful and becomes more accurate for larger values of α. It may also be noted that although the mean GSM voice queue length is inversely proportional to α, the mean waiting GSM voice waiting time is not heavily dependent on the value of α. This is due to the fact that the GSM voice arrival rate also reduces when we increase α, while keeping ρ_v constant.

Based on these results we can conclude that for low to medium traffic loading of GPRS data, the impact on GSM voice waiting time in queue is quite low (e.g. for ρ_d < 3, EW_V is less than 50 msec, see figure 7). This implies that the delayed release of GPRS packets can be conveniently implemented in GSM/GPRS hence simplifying the implementation of the system as compared to the immediate release scenario. Also, in comparison with the protocol studied in [7] termed as 'without channel de-allocation', the delayed release model achieves the low voice blocking probability (as shown in [7]) as well as low voice queuing delay and seems to be the more suitable protocol.

Finally, from the results we can also conclude that the decomposition approximation can be usefully applied to the study of complex multi-dimensional Markov chains where transition rates of different orders of magnitude exist.

[2] Here ρ_v is chosen such that the voice blocking probability is approximately 15% in both the cases.

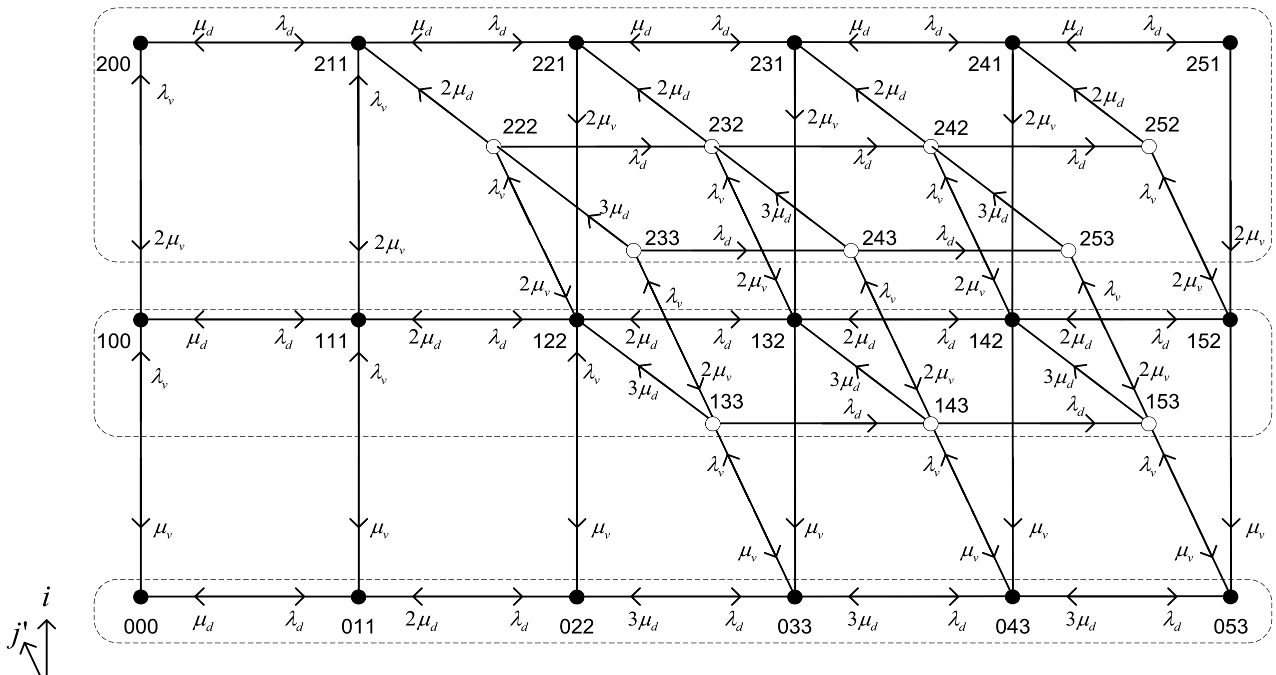

Figure 1: State transition diagram for GPRS 3(2,1)5 Non-preemptive priority

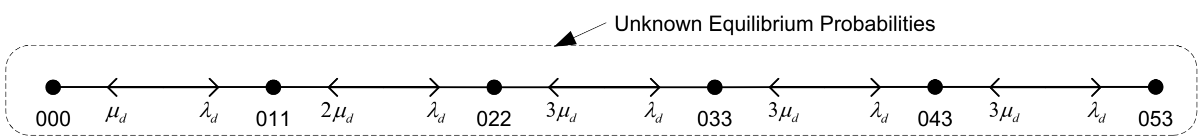

Figure 2.0.A : Step 0, part A of GPRS 3(2,1)5 example.

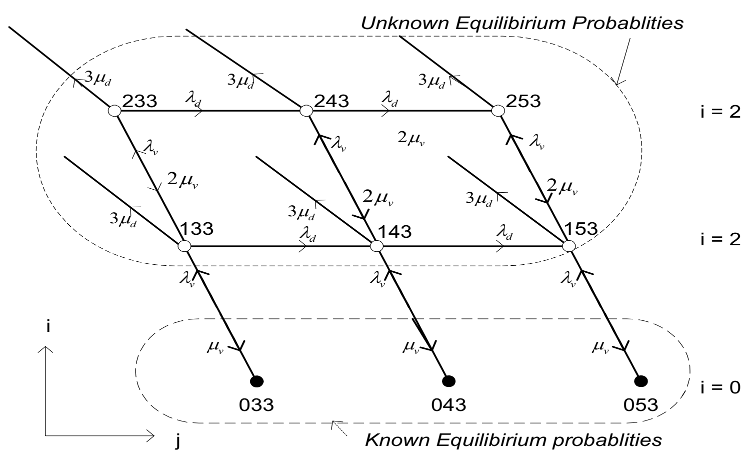

Figure 2.0.B : Step 0, part B of GPRS 3(2,1)5 example.

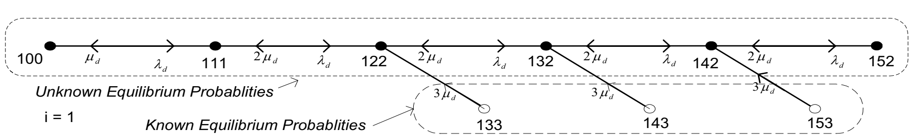

Figure 2.1.A : Step 1, part A of GPRS 3(2,1)5 example.

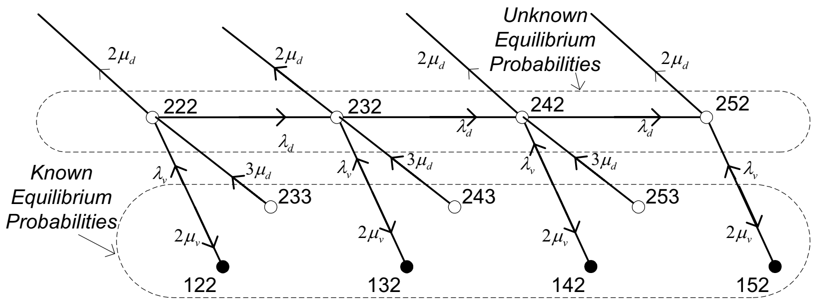

Figure 2.1.B : Step 1, part B of GPRS 3(2,1)5 example.

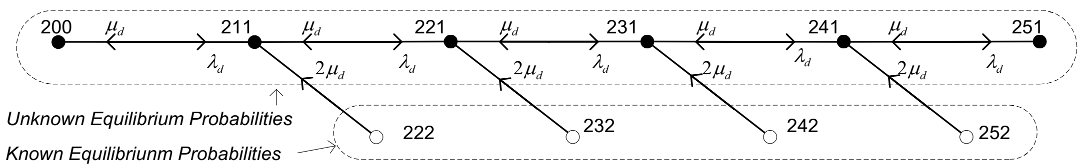

Figure 2.2.A : Step 2, part A of GPRS 3(2,1)5 example.

Figure 2: Decomposition of GPRS 3(2,1)5, steps (0.A-2.A).
Note: In all the figures 2.0.A-2.2.A, only states and transitions that are used in the calculation of the unknown equilibrium probabilities are shown.

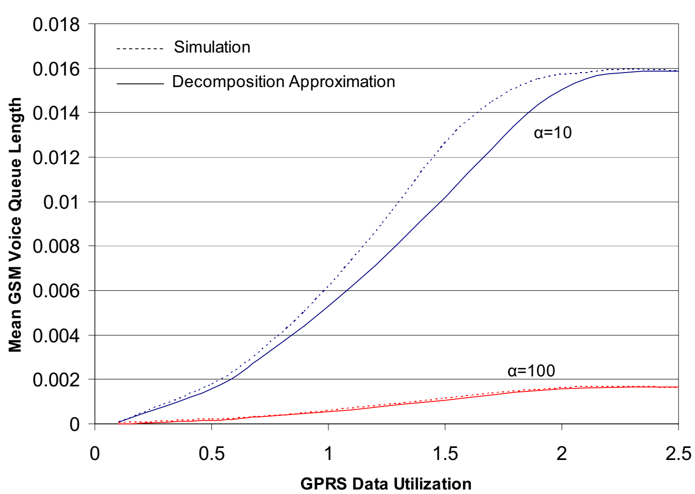

Figure 3: Decomposition & simulation: EQ_V for GPRS 2(1,1)30, α =10, 100.

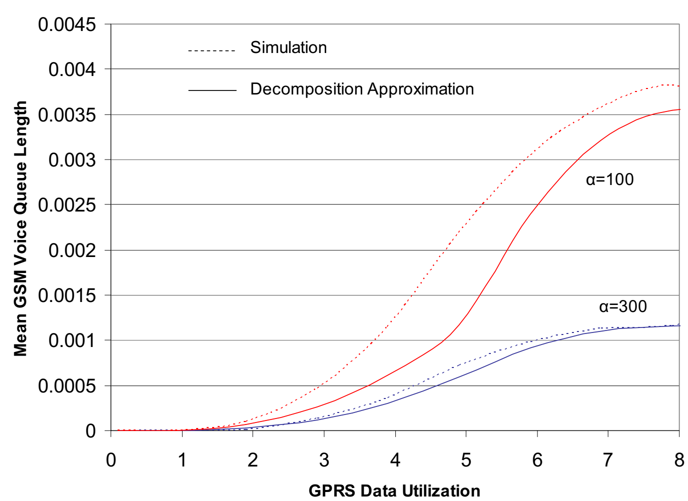

Figure 4: Decomposition & simulation: EQ_V for GPRS 8(4,4)100, α =100, 300.

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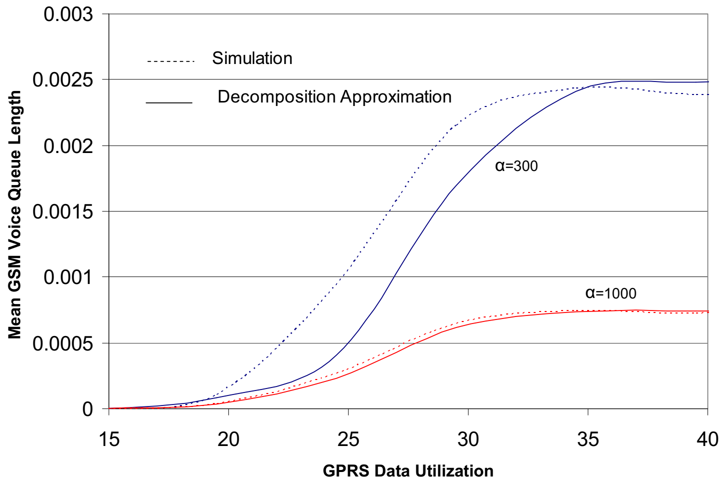

Figure 5: Decomposition & simulation: EQ_V for 50(25,25)100, α =300, 1000.

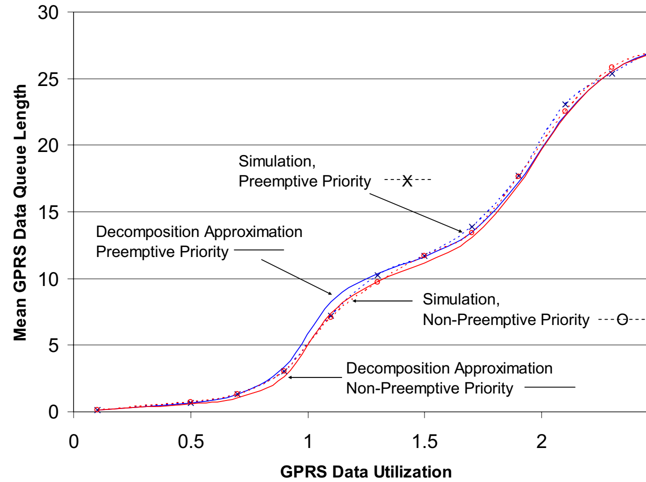

Figure 6: Preemptive & Non-preemptive, Decomposition & simulation: EQ_D for GPRS 2(1,1)30, α =300.

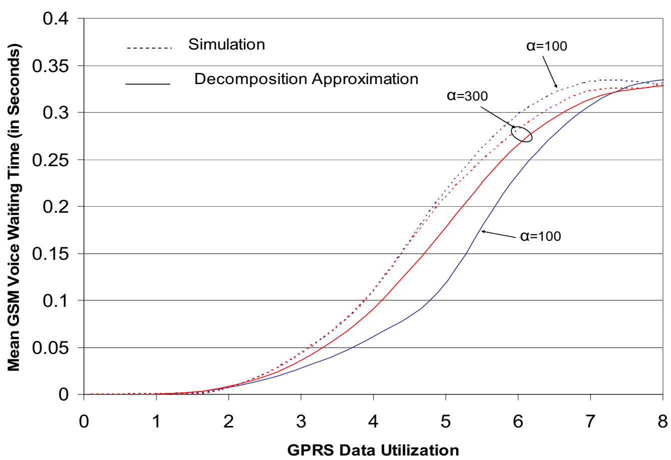

Figure 7: Decomposition & simulation: EW_V for GPRS 8(4,4)100, α =100, 300.

Appendix A. Detailed Analysis

In section A.1 we start by obtaining an exact solution to the queued states S_Q in terms of the non-queued states S_N, using a recursive solution given by Proposition 1 below (this is used in parts B of Algorithm 1 below). The queued states S_Q, we recall, are those in which GSM voice calls are queued while competing with Non-preemptive priority GPRS data packets which are occupying the shared channels. The non-queued states S_N are those in which the above is not true. In section A.2 we formulate and solve the general decomposition problem for $V(t) > 0$ for states in S, assuming that a solution to some of the queued states S_Q is already known. Finally in section A.3 we develop Algorithm 1, based on Proposition 1 and the decomposition solutions mentioned above, to approximate all the equilibrium probabilities of the general Non-preemptive problem.

A.1 Relating $P_{i,j,j'}$ to $P_{i-1,j,j'}$

We use the same notation as in equation 2.4 of [2] to describe the transition rates of the Markov chain $\{V(t), D(t), D'(t)\}$. Thus the transition rate from state $x \in S$ to another state $y \in S$ is given as $q(x;y)$. The following equations describe all the possible transitions assuming both $x, y \in S$.

$$
\begin{aligned}
&q(i,j,j';i+1,j,j') = \lambda_v^i && \\
&q(i,j,j';i-1,j,j') = \mu_v^i && \text{if } j'=j \\
&q(i,j,j';i-1,j,j'+1) = \mu_v^i && \text{if } j'<j \\
&q(i,j,j';i,j+1,j') = \lambda_d^j && \text{if } j' \geq N-i \qquad \text{(A.0)} \\
&q(i,j,j';i,j+1,j'+1) = \lambda_d^j && \text{if } j' < N-i \\
&q(i,j,j';i,j-1,j'-1) = \mu_d^{j'} && \text{if } j'=j \text{ or } V_Q(t)>0 \\
&q(i,j,j';i,j-1,j') = \mu_d^{j'} && \text{if } j'>j \text{ or } V_Q(t)=0
\end{aligned}
$$

Here we have used the shorthand notation λ_v^i and μ_v^i to denote the rates of departure from GSM voice state $V(t)=i$, due to a GSM voice arrival or departure respectively. Similarly λ_d^i and μ_d^i are the corresponding values for GPRS data. The non-zero transition rates are then given by:

$$\begin{aligned} &\lambda_v^i = \lambda_v && \text{for } 0 \le i < N_v \\ &\mu_v^i = i\mu_v && \text{for } 0 < i \le N_v \\ &\lambda_d^j = \lambda_d && \text{for } 0 \le j < M \\ &\mu_d^{j'} = j'\mu_d && \text{for } 0 < j' \le N \end{aligned} \qquad \text{(A.1)}$$

We now define the terms $F_{i,j,j'}$ and $R_{i,j,j'}$ for $(i,j,j') \in S_Q$ with the help of figure A1:

$F_{i,j,j'} \equiv$ Total *probability flux* into state $(i,j,j') \in S_Q$ other than from $(i-1,j,j')$ or $(i+1,j,j')$. If we define $A \equiv P_{i,j-1,j'}\lambda_d$, $B \equiv P_{i,j+1,j'+1}(j'+1)\mu_d$, $C \equiv N-(i-1)$, then using (A.1), and figures 1 and A2 it can be verified that $F_{i,j,j'}$ is given as follows:

$$F_{i,j,j'} = \begin{cases} A+B & \text{for} \quad j > C \quad \text{and} \quad (j' < N \quad \text{and} \quad j < M) \\ B & \text{for} \quad j = C \quad \text{and} \quad (j' < N \quad \text{and} \quad j < M) \\ A & \text{for} \quad j > C \quad \text{and} \quad (j' = N \quad \text{or} \quad j = M) \\ 0 & \text{for} \quad j = C \quad \text{and} \quad (j' = N \quad \text{or} \quad j = M). \end{cases}$$

The boundary conditions above can be explained as follows. For $(i,j,j') \in S_Q$, if $j=C$ then $i+j = i+N-(i-1) = N+1$. This implies that one GSM voice call is queued and all the GPRS data packets are being served. In this case $j'=j$ and the state $(i,j-1,j')$ does not exist and hence $A=0$. Similarly when $j'=N$ or $j=M$, state $(i,j+1,j'+1)$ does not exist and $B=0$.

$R_{i,j,j'} \equiv$ Total *rate* out of state (i,j,j'). This is given by (see figure A1):

$$R_{i,j,j'} = \lambda_d^j + \mu_d^{j'} + \lambda_v^i + \mu_v^i. \qquad \text{(A.2)}$$

Now the following proposition gives an exact relationship between $P_{i,j,j'}$ and $P_{i-1,j,j'}$ using the above definition of $F_{i,j,j'}$ and $R_{i,j,j'}$:

Proposition 1. *For $(i,j,j') \in S_Q$, $P_{i,j,j'}$ satisfies the following recursive relationship:*

$$P_{i,j,j'} = \Gamma_{i-1,j,j'} + P_{i-1,j,j'}\Theta_{i-1,j,j'} \quad i = 1,\ldots,N_v, \quad \textit{(A.3)}$$

with $\Gamma_{i-1,j,j'}$ and $\Theta_{i-1,j,j'}$ also given recursively by

$$\Gamma_{i-1,j,j'} = \begin{cases} \dfrac{F_{i,j,j'} + \mu_v^{i+1}\Gamma_{i,j,j'}}{R_{i,j,j'} - \mu_v^{i+1}\Theta_{i,j,j'}} & 1 \le i \le N_v \\ 0 & i > N_v \end{cases} \quad \textit{(A.4)}$$

$$\Theta_{i-1,j,j'} = \begin{cases} \dfrac{\lambda_v^{i-1}}{R_{i,j,j'} - \mu_v^{i+1}\Theta_{i,j,j'}} & 1 \le i \le N_v \\ 0 & i > N_v \end{cases} \quad \textit{(A.5)}$$

λ_v^i and μ_v^i are given by (A.1).

The above Proposition can be proved by writing the local balance equation at (i,j,j') and then using induction on i.

We note that Proposition 1 has written $P_{i,j,j'}$ for $(i,j,j') \in S_Q$ such that it only depends on the following three adjacent states $(i-1,j,j')$, $(i,j-1,j')$, $(i,j+1,j'+1)$. We also note that proposition 1 gives an exact relationship between these states.

A.2 Decomposition of Aggregate state $V(t)=i$

We now formulate and solve the decomposition problem for states in the aggregate state $V(t)=i$. Transitions to and from such a state (i,j,j') are shown in figure A1. The set of such states consists of states in S_N whose equilibrium probabilities need to be calculated, and states in S_Q whose equilibrium probabilities have already been approximated in an earlier step. We can solve for the unknown equilibrium probabilities by writing the local balance equations. The balance equations are written down by equating the probability flux in each direction across the cuts shown in figure A2 by the dotted lines.

$$\begin{aligned} P_{i,0,0}\lambda_d &= P_{i,1,1}\mu_d \\ P_{i,1,1}\lambda_d &= P_{i,2,2}2\mu_d \\ &\vdots \\ P_{i,N-i-1,N-i-1}\lambda_d &= P_{i,N-i,N-i}(N-i)\mu_d \\ P_{i,N-i,N-i}\lambda_d &= P_{i,N-i+1,N-i}(N-i)\mu_d + \mu_d f_{i,N-i+1} \\ P_{i,N-i+1,N-i}\lambda_d &= P_{i,N-i+2,N-i}(N-i)\mu_d + \mu_d f_{i,N-i+1} + \mu_d f_{i,N-i+2} \\ &\vdots \\ P_{i,M-1,N-i}\lambda_d &= P_{i,M,N-i}(N-i)\mu_d + \mu_d \textstyle\sum_{j=N-i+1}^{M} f_{i,j} \end{aligned}$$
$$P_{i,0,0} + P_{i,1,1} + \cdots + P_{i,M,N-i} = p_v(i) - p_Q(i). \qquad \text{(A.7)}$$

Here we have defined

$\mu_d f_{i,j} \equiv$ Probability flux from state $(i,j,N-i+1) \in S_Q$ into state $(i,j-1,N-i) \in S_N$

$$= (N-i+1)\mu_d P_{i,j,N-i+1}, \qquad \text{(A.8)}$$

and

$$p_Q(i) \equiv \sum_{\forall j,j' \ \ s.t. \ \ (i,j,j') \in S_Q} P_{i,j,j'}. \qquad \text{(A.9)}$$

This gives us M unknowns, i.e. $P_{i,0,0}, P_{i,1,1}, \cdots, P_{i,M,N-i}$, which can be solved for. In fact a closed form equation for these can be written down. It can be shown that a closed form solution to the above equations is given by the following:

If we define $\overline{P}_{i,j}$ as the non-queued states:

$$\overline{P}_{i,j} \equiv \begin{cases} P_{i,j,j} & 0 \le j \le N-i \\ P_{i,j,N-i} & N-i+1 \le j \le M \end{cases} \tag{A.10}$$

Then

$$\overline{P}_{i,0} = \frac{p_v(i) - p_Q(i) + \sum_{j=N-i+1}^{M} \sum_{l=0}^{j-(N-i+1)} \left(\frac{\rho_d}{N-i}\right)^{j-(N-i+1+l)} \sum_{k=N-i+1}^{N-i+l+1} \frac{f_{i,k}}{N-i}}{\sum_{j=0}^{N-i} \frac{\rho_d^j}{j!} + \frac{\rho_d^{N-i}}{(N-i)!} \sum_{k=1}^{M-(N-i)} \left(\frac{\rho_d}{N-i}\right)^k} \tag{A.11}$$

$$\overline{P}_{i,j} = \begin{cases} \overline{P}_{i,0} \frac{\rho_d^j}{j!} & 1 \le j \le N-i \\ \overline{P}_{i,0} \frac{\rho_d^{N-i}}{(N-i)!} \left(\frac{\rho_d}{N-i}\right)^{j-(N-i)} - \sum_{l=0}^{j-(N-i+1)} \left(\frac{\rho_d}{N-i}\right)^{j-(N-i+1+l)} \sum_{k=N-i+1}^{N-i+l+1} \frac{f_{i,k}}{N-i} & N-i+1 \le j \le M \end{cases} \tag{A.12}$$

A.3 Algorithm for Solution to Non-preemptive Problem

In section A.1 we developed a recursive equation which showed how the equilibrium distribution of states in S_Q could be calculated from those of states in S_N, using intermediate parameters $\Theta_{i,j,j'}$ and $\Gamma_{i,j,j'}$. In section A.2 we showed how the decomposition solution to the states in each aggregate state could be found. We now give the algorithm which calculates all the equilibrium probabilities in the correct order as also required by Proposition 1. We of course also calculate the intermediate parameters $\Theta_{i,j,j'}$ and $\Gamma_{i,j,j'}$.

The algorithm proceeds in steps 0 through N_v. In step i, part A of the algorithm finds, using the decomposition solution of section A.2, the equilibrium distribution of all the states in $V(t)=i$, which belong to S_N. Here it uses the solution to the equilibrium probabilities of states in $V(t)=i$, belonging to S_Q, which were obtained in an earlier step. Note that in step 0, only states in S_N exist (see figure 1).

In step i, part B of the algorithm finds the equilibrium probabilities of all the states in S_Q, with $V(t)>i, j \ge N-i$, and $j'=N-i$, by using the exact dependence of these on the equilibrium probabilities of states in $V(t)=i$ and states with $j' \ge N-i+1$ both belonging to S_N, using Proposition 1.

Algorithm 1. The following algorithm is used to calculate the *decomposition approximation* $P_{i,j,j'}$, to the equilibrium probability distribution of the Markov chain formed by $\{V(t), D(t), D'(t)\}$ whose transitions are given by (A.0). References to 'Steps' in the algorithm, refer to Step i', which consists of Parts A and B.

For $i'=0$ *to* $N_v - 1$ *(Step* i'*)*

Part A.

Calculate $P_{i',.,.}$ *for* $(i,.,.) \in S_N$ *by using the decomposition technique given by equations (A.11), (A.12) of section A.2.*

Part B.

Let $j' = N - i'$

For $j = j'$ *to* M

For $i = N_v - 1$*, step* -1*, to* i'

Calculate $\Theta_{i,j,j'}, \Gamma_{i,j,j'}$ *using equations (A.4), (A.5)*

Next i

For $i = i'+1$ *to* N_v

Calculate $P_{i,j,j'}$ *using equation (A.3)*

Next i

Next j

Next i'

(Step N_v*) Calculate* $P_{N_v,.,.}$ *for* $(N_v,.,.) \in S_N$ *by using equations (A.11), (A.12).*

Let us now give expressions for our performance criterion. The mean GPRS data queue length is simply

$$EQ_D = \sum_{\forall (i,j,j') \in S} j P_{i,j,j'},$$
$$= \sum_{i=0}^{N_v} \sum_{j=0}^{M} j \sum_{j'=\min(j,N-i)}^{\min(j,N)} P_{i,j,j'}. \tag{A.13}$$

The mean GPRS data blocking probability is

$$P_{BD} = \sum_{\forall (i,M,j') \in S} P_{i,M,j'},$$
$$= \sum_{i=0}^{N_v} \sum_{j'=\min(M,N-i)}^{\min(M,N)} P_{i,M,j'}. \tag{A.14}$$

The mean number of queued GSM voice calls is

$$EQ_V = \sum_{\forall (i,j,j') \in S_Q} \max\{0, i-(N-j')\} P_{i,j,j'} \tag{A.15}$$

The mean GSM voice blocking probability was already given by (3.1). Finally, the mean waiting time of all non-blocked GSM voice calls is given by: $EW_V = EQ_v / (\lambda_v (1 - P_{BV}))$.

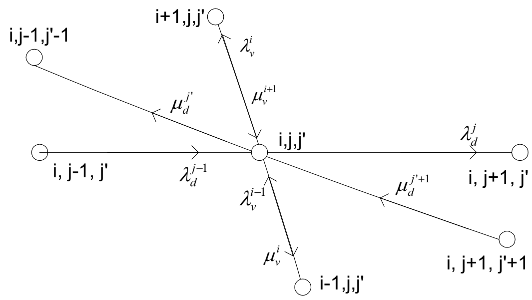

Figure A1: Transitions to and from state $(i, j, j') \in S_N$

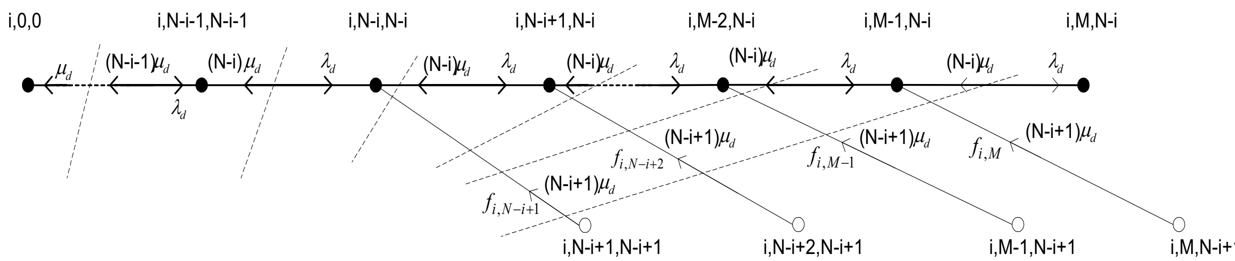

Figure A2: Transitions in Non-preemptive Markov chain for states in S_Q

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